

## Relationship between decision, optimization, and construction problems

We define Algorithm  $\mathcal{C}$  to find a maximum matching, as follows. The input is graph  $G = (V, E)$ . If  $E = \emptyset$ , we return  $\emptyset$ . Otherwise, let  $E = \{e_1, e_2, \dots, e_m\}$ , and let  $k = \mathcal{B}(G)$ .

- ▶ Let  $G^*$  be a copy of  $G$
- ▶ For  $i = 1$  up to  $m$  DO
  - ▶ Let  $G'$  be the graph obtained by deleting edge  $e_i$  (but not the endpoints of  $e_i$ ) from  $G^*$ .
  - ▶ If  $\mathcal{A}(G', k) = \text{YES}$ , then set  $G^* := G'$ .
- ▶ Return the edge set  $E(G^*)$  of  $G^*$ .

It is easy to see that  $\mathcal{C}$  calls  $\mathcal{B}$  once, calls  $\mathcal{A}$  at most  $m$  times, and does at most  $O(m)$  other operations. Hence the running time satisfies the required bounds.

What about accuracy?