Relationship between decision, optimization, and construction problems

To solve the optimization problem, we define Algorithm $\mathcal{B}$ as follows.
The input is graph $G = (V, E)$. If $E = \emptyset$, we return 0. Else, we do the following:

- For $k = |E|$ down to 1, DO
  - If $\mathcal{A}(G, k) = YES$, then $Return(k)$

It is easy to see that

- $\mathcal{B}$ is correct,
- that $\mathcal{B}$ calls $\mathcal{A}$ at most m times
- that $\mathcal{B}$ does at most $O(m)$ additional steps.

Hence $\mathcal{B}$ satisfies the desired properties.