Things to think about

- Suppose G is a simple graph that has a maximum matching of size k and a minimum vertex cover of size k'. Prove that $k' \geq k$.
- Prove that every tree can be 2-colored.
- Prove that every tree with at least two three vertices has a sibling pair of leaves (where two leaves are siblings if they share a neighbor).
- Come up with a simple algorithm to find a *maximal matching* (i.e., a matching that cannot be enlarged by adding another edge) in a graph, and analyze its running time.
- Show how having an algorithm to compute the chromatic number in a graph can be used to find an optimal vertex coloring for a graph, with only a polynomial number of calls to the algorithm.