Finding the largest matching in a graph

Recall k is the size of a maximum matching in input graph G, with edge set $\{e_1, e_2, \ldots, e_m\}$, $m \geq 1$.

- Let G^* be a copy of G
- For $i = 1$ up to m DO
  - Let G' be the graph obtained by deleting edge e_i (but not the endpoints of e_i) from G^*.
  - If $\mathcal{A}(G', k) = YES$, then set $G^* = G'$.

  Return the edge set of G^*.

Theorem: The edge set E^* of G^* is a matching in G.

Proof (by contradiction): If not, then E^* has at least two edges e_i and e_j (both from E) that share an endpoint. Let E_0 be a maximum matching in G^*; hence E_0 is a maximum matching for G. Note that E_0 cannot include both e_i and e_j. Suppose (*w.l.o.g.*) $e_i \notin E_0$. During the algorithm, we checked whether a graph G' that did not contain e_i had a matching of size k. Since we did not delete e_i, this means the answer was NO. But the edge set of that G' contains the matching E_0, which means G' has a matching of size k, yielding a contradiction.