Describing a real world problem

You want to partition the set of people into subsets so that every two people in any subset are friends, and make the number of subsets as small as possible.

Solution: The graph $G = (V, E)$ is defined by

- V is the set of people in the class
- E contains (v, w) if and only if v and w are friends.

We are looking for a partition of V into a small number of sets so that every one of the sets is a clique (which means that all pairs of vertices in any of the sets are adjacent).

In other words, we want to write $V = V_1 \cup V_2 \cup \ldots \cup V_k$, where V_i is a clique in G and where k is minimized.

Questions to you:

- Does a solution always exist?
- Does it have to be unique?
- What graph problem does this resemble?
- How hard is it to solve the problem?