Formulating real world problems as graph problems

You want to find a set of people in this class so that everyone in the class is friends with someone in the set, and the set is as small as possible.

The graph G = (V, E) is defined by:

- Let V denote all the people in the class.
- ▶ Put an edge between *v* and *w* if *v* and *w* are friends.

We are looking for the smallest $V_0 \subseteq V$ such that $\forall v \in V \setminus V_0$, $\exists w \in V_0$ such that $(v, w) \in E$.

Questions:

- Does a solution always exist?
- What graph problem is this?
- ▶ How hard is it to solve this problem?