Formulating real world problems as graph problems

Consider the following problems, all based on the set of people in this class, and under the assumption where you know who is friends with whom (and you assume friendship is symmetric).

- 1. You want to find a set of people in this class so that everyone in the class is friends with someone in the set, and the set is as small as possible.
- 2. You want to partition the set of people into subsets so that every two people in any subset are friends, and make the number of subsets as small as possible.
- 3. You want to pair people off in the class so that they will study together. You want to have as large a number of people able to be in study groups, but you have the following rules: study groups must be pairs of friends and no person can be in two study groups.
- 4. You want to have a party and invite as many people as you can to it, subject to (a) they are all your friends and (b) they all friends with each other.