

VERTEX COLORING

Definition: A proper **vertex coloring** of a graph is an assignment of colors to the vertices so that no two adjacent vertices get the same color. Thus, a proper vertex coloring is a function $f : V \rightarrow \{1, 2, \dots, k\}$ such that $f(v) = f(w) \Rightarrow (v, w) \notin E$.

Decision problem:

- ▶ Input: Graph $G = (V, E)$ and integer k
- ▶ Question: Does G have a proper k -coloring?

Optimization problem: Find the minimum number of colors needed to vertex-color an input graph G . (This number is called the **chromatic number**.)

Construction problem: Find a vertex coloring of input graph G that uses the minimum number of colors.

Note: Vertex Coloring is NP-complete (one of Karp's original 21 problems)