

## Proving the DP algorithm for Fibonacci is correct

To prove that  $FIB[N + 1] = F(N + 1)$ , note that  $N \geq 2$  so  $N + 1 \geq 3$ .

Hence  $FIB[N + 1] = FIB[N] + FIB[N - 1]$ , by the DP algorithm.

By the inductive hypothesis  $FIB[N] = F(N)$  and  $FIB[N - 1] = F(N - 1)$ , and so  $FIB[N + 1] = F(N) + F(N - 1)$ .

Hence,  $FIB[N + 1] = F(N + 1)$ , by the definition of the Fibonacci numbers.

Since  $N$  was arbitrary, by the Principle of Mathematical Induction,  $FIB[N] = F(N)$  for all non-negative integers  $N$ .