Proving the DP algorithm correct

Let $F(n)$ be the n^{th} Fibonacci number, defined recursively by

- $F(1) = F(2) = 1$ and
- $F(n) = F(n-1) + F(n-2)$ for $n \geq 3$

We prove that $F(n) = FIB[n]$ by strong induction on n.

Let $P(N)$ denote the statement $\forall n \in \mathbb{Z}^+, n \leq N, FIB[n] = F(n)$.

By definition, $FIB[n] = F(n)$ for $n = 1, 2$, so the two base cases are true.

We have shown $P(1)$ and $P(2)$ is true (our base cases).

Our Strong Inductive Hypothesis is that $P(N)$ is true for some arbitrary $N \geq 2$.

We wish to prove that $P(N+1)$ is true.

In other words, we wish to prove that $FIB[N+1] = F(N+1)$.