Proving f is onto

Recall that we need to prove that f is a bijection from $\mathbb{Z}$ to $\mathbb{N}$, where

- $f(x) = 2x$ when $x \geq 0$
- $f(x) = 2|x| - 1$ when $x < 0$

To prove that f is onto we need to show that for any $b \in \mathbb{N}$ there is some $a \in \mathbb{Z}$ such that $f(a) = b$.

Case: b is odd. Then $b = 2x + 1$ for some $x \in \mathbb{Z}^+$.

Let $a = -(x + 1)$. Then

$$f(a) = 2|a| - 1 = 2(x + 1) - 1 = 2x + 1 = b$$

Case: b is even. Then $b = 2x$ for some $x \in \mathbb{Z}^{\geq 0}$. Then

$$f(x) = 2x = b$$

Hence f is onto.