Proving f is $1-1$

Recall $f : \mathbb{Z}$ to $\mathbb{N}$ is defined by

- $f(x) = 2x$ when $x \geq 0$
- $f(x) = 2|x| - 1$ when $x < 0$

We prove that f is $1-1$ by contradiction. If f is not $1-1$, then $\exists\{a, b\} \subset \mathbb{Z}$ such that $f(a) = f(b)$.

Since $f(x)$ is odd if and only if x is negative, it must be that a and b are both negative or both non-negative.