Proof that $|\mathbb{Z}| = |\mathbb{N}|$

We prove that $|\mathbb{Z}| = |\mathbb{N}|$ by establishing a bijection from $\mathbb{Z}$ to $\mathbb{N}$.

We will send the non-negative integers to the even natural numbers, and the negative integers to the odd natural numbers.

- $f(x) = 2x$ when $x \geq 0$
- $f(x) = 2|x| - 1$ when $x < 0$

It is clear that f maps integers to natural numbers. To complete the proof:

- We need to prove that f is $1 - 1$
- We need to prove that f is onto