Infinite Sets

A set X is **infinite** if there does not exist any $n \in Z$ so that $|X| = n$.

Formal definition: A set X is infinite if $\exists Y \subset X$ (i.e., Y is a proper subset of X) and a 1-1 function $f : X \to Y$.

Examples:

- Let E denote the set of even integers and let $f : \mathbb{Z} \to E$ be defined by $f(x) = 2x$.
- Let $g : \mathbb{Z}^+ \to \mathbb{Z}^{\geq 5}$ be defined by $g(x) = x + 5$

Each of these is a 1-1 function from a set A to a proper subset of A. Hence the set A is infinite.

We say that $|X| \leq |Y|$ if there is a 1-1 function $g : X \to Y$.