

Using Cantor-Schroeder-Bernstein Theorem

For example, to prove $|\mathbb{N}| = |\mathbb{Z}|$, we can write

- ▶ $f : \mathbb{N} \rightarrow \mathbb{Z}$, where $f(x) = x$
- ▶ $g : \mathbb{Z} \rightarrow \mathbb{N}$, where
 - ▶ $g(x) = 2x$ if $x \geq 0$
 - ▶ $g(x) = 2|x| + 1$ if $x < 0$

It's easy to see that f and g are both 1-1, so by the Cantor-Schroeder-Bernstein theorem, $|\mathbb{N}| = |\mathbb{Z}|$.