

General properties

- ▶ If $|X| \leq |Y|$ and Y is countable, then X is countable (recall that $|X| \leq |Y|$ means there is a 1-1 function from X to Y).
- ▶ If X_1, X_2, \dots, X_k are each countable, then $\prod_i X_i$ is countable.
- ▶ If X_1, X_2, \dots, X_k are each countable, then $\cup_i X_i$ is countable.

Hence $\mathbb{Z} \times \mathbb{Z}$ and \mathbb{Q} are both countable.