When is $A \times B$ countable?

Consider the infinite matrix $M[i,j]$ where $M[i,j]$ corresponds to the ordered pair (a_i, b_j).

Consider the enumeration of the set $A \times B$, given by going down short diagonals (right to left, decreasing):

- $M[1,1]$
- $M[1,2], M[2,1]$
- $M[1,3], M[2,2], M[3,1]$
- $M[1,4], M[2,3], M[3,2], M[4,1]$
- etc.

Note that every element of $A \times B$ appears at some finite index, and so enumeration defines a bijection between the elements of $A \times B$ and Z^+.

Hence if A and B are countable, then $A \times B$ is countable.