We prove $\mathbb{P}(\mathbb{N})$ is uncountable using a diagonalization argument.

Consider the infinite matrix representing $\mathbb{P}(\mathbb{N})$.

By construction, every subset of $\ensuremath{\mathbb{N}}$ is represented by some row in the matrix.

Consider the set Y defined by $j \in Y$ if and only if $M_{j,j} = 0$.

Note that Y is a subset of \mathbb{N} .