

Using L'Hôpital's Rule

Let $f(n) = n$ and $g(n) = e^n$. We want to prove that f is $O(g)$.

Proof:

To prove that f is $O(g)$, it must be that $\exists C, k > 0$ such that $\forall n > K, |f(n)| < C|g(n)|$.

Equivalently, $\forall n > K, \left| \frac{f(n)}{g(n)} \right| < C$.

We try to compute $\lim_{n \rightarrow \infty} \left| \frac{f(n)}{g(n)} \right|$, but it isn't easy to see what it is.

Can we use L'Hôpital's Rule?