

More tricks

Theorem: Suppose $f : \mathbb{N} \rightarrow \mathbb{R}$ and $g : \mathbb{N} \rightarrow \mathbb{R}$ and

$$\lim_{n \rightarrow \infty} \left| \frac{f(n)}{g(n)} \right| = C$$

for some constant C . Then f is $O(g)$!

Proof: When $\lim_{n \rightarrow \infty} \left| \frac{f(n)}{g(n)} \right| = C$, then $\exists k$ such that for all $n > k$, $\left| \frac{f(n)}{g(n)} \right| < C + 1$.

Hence, $|f(n)| < (C + 1)|g(n)|$ for all $n > k$.

Hence, f is $O(g)$.