

Why does Kruskal's algorithm work?

Remember $e = (x, y) \in E(T^*) \setminus E(T)$

Since e is not in $E(T)$, it must be that when e is considered it would create a cycle if added to T .

If we add e to T we obtain a cycle γ and T has a path P from x to y given by $x, v_1, v_2, \dots, v_k, y$.

Because $e \notin E(T)$, it must be that $w(e) > w(e')$ for all $e' \in E(P)$ (otherwise Kruskal's algorithm would have added e).