

# Floyd-Warshall Algorithm

After we compute all entries of the matrix  $M$  with  $k = 0, 1, \dots, K - 1$ , can we compute the entries with  $k = K$ ?

Consider a shortest path  $P$  from  $v_i$  to  $v_j$  with  $MAX(P) \leq K$ .

Cases:

- ▶  $P$  satisfies  $MAX(P) \leq K - 1$ . Then  $Cost(P) = M[i, j, K - 1]$ .
- ▶  $P$  satisfies  $MAX(P) = K$ .

Hence  $v_K \in P$ . Analyzing this is a bit more complicated, but we will show the path  $P$  satisfies

$$Cost(P) = M[i, K, K - 1] + M[K, j, K - 1].$$