

# Floyd-Warshall Algorithm

Consider a shortest path  $P$  from  $v_i$  to  $v_j$  with  $MAX(P) \leq 1$ .

Cases:

- ▶  $P$  is a single edge  $e$ , and so  $Cost(P) = w(e) = M[i, j, 0]$ .

- ▶  $P$  has an internal vertex, which must be  $v_1$ .

Hence  $P$  has two edges,  $(v_i, v_1)$  and  $(v_1, v_j)$ .

Then  $Cost(P) = w(v_i, v_1) + w(v_1, v_j)$ .

Note that  $w(v_i, v_1) = M[i, 1, 0]$  and  $w(v_1, v_j) = M[1, j, 0]$ .

Hence  $M[i, j, 1] = \min\{M[i, j, 0], M[i, 1, 0] + M[1, j, 0]\}$ .